

Linear Programming

1. WELLTYPE Manufacturing Company produces three types of typewriters; Manual typewriters, Electronic typewriters, and Deluxe Electronic typewriters. All the three models are required to be machined first and then assembled. The time required for the various models are as follows :

Types	Machine Time (in hours)	Assembly Time (in hours)
Manual Typewriter	15	4
Electronic Typewriter	12	3
Deluxe Electronic Typewriter	14	5

The total available machine time and assembly time are 3,000 hours and 1,200 hours respectively. The data regarding the selling price and variable costs for the three types are :

	Manual	Electronic	Deluxe Electronic
Selling Price (₹)	4,100	7,500	14,600
Labour, material and other variable costs (₹)	2,500	4,500	9,000

The company sells all the three types on credit basis, but will collect the amounts on the first of next month. The labour, material and other variable expenses will have to be paid in cash. This company has taken a loan of ₹ 40,000 from a co-operative bank and this company will have to repay it to the bank on 1st April, 2004. The TNC Bank from whom this company has borrowed ₹ 60,000 has expressed its approval to renew the loan.

The Balance Sheet of this Company as on 31.3.04 is as follows :

Liabilities		Assets	
Equity Share capital	1,50,000	Land	90,000
Capital Reserve	15,000	Building	70,000
General Reserve	1,10,000	Plant & Machinery	1,00,000
Profit & Loss a/c	25,000	Furniture & Fixtures	15,000
Long term loan	1,00,000	Vehicles	30,000
Loan from TNC Bank	60,000	Inventory	5,000
Loan from Co-op. Bank	40,000	Receivables	50,000
		Cash	1,40,000
Total	<u>5,00,000</u>	Total	<u>5,00,000</u>

The company will have to pay a sum of ₹ 10,000 towards the salary form top management executives and other fixed overheads for the month. Interest on long term loans is to be paid every month at 24% per annum. Interest on loans from TNC and Co-operative Banks may be taken to be ₹ 1,200 for the month.

The company has promised to deliver 2 Manual typewriters and 8 Deluxe Electronic typewriters to one of its valued customers next month. Also make sure that the level of operations in this company is subject to the availability of cash next month. This company will also be able to sell all three types of typewriters in the market. The senior Manager of this company desires to know as to how many units of each typewriter must be manufactured in the factory next month so as to maximise the profits of the company.

Formulate this as a linear programming problem. The formulated problem need not be solved.

2. The most recent audited summarised Balance Sheet of Stop and Shop Financial Services is given below :

Balance Sheet as on March 31, 2004					
Liabilities	(` in lakhs)	Assets	(` in lakhs)		
Equity Share Capital	65	Fixed Assets :			
Reserves & Surplus	110	- Assets on Lease			
		(original cost: ` 550 lakhs)			375
Term Loan from IFCI	80	- Other Fixed Assets			50
Public Deposits	150	Investments (on wholly owned subsidiaries)			20
Bank Borrowings	147				
Other Current Liabilities	50	Current Assets :			
		- Stock on Hire			80
		- Receivables			30
		- Other Current Assets			35
		Miscellaneous expenditure (not written off)			12
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The Company intends to enhance its investment in the lease portfolio by another ` 1,000 lakhs. For this purpose it would like to raise a mix of debt and equity in such a way that the overall cost of raising additional funds is minimised. The following constraints apply to the way the funds can be mobilised :

Total debt divided by net owned funds, cannot exceed 10.
Amount borrowed from financial institutions cannot exceed 25% of the net worth.
Maximum amount of bank borrowings cannot exceed three times the net owned funds.
The company would like to keep the total public deposit limited to 40% of the total debt.

The post-tax costs of the different sources of finance are as follows :

Equity	:	25%	Term Loans	:	8.5%
Public Deposits	:	7%	Bank Borrowings	:	10%

Formulate the funding problem as a L.P.P.

Note :

Total Debt = Term Loans from Financial Institutions + Public Deposits + Bank Borrowings
Net Worth = Equity Share Capital + Reserves & Surplus
Net owned Funds = Net worth - Miscellaneous Expenditures

3. The final simplex table for the problem is given below :

Maximize $Z = 3x_1 + 4x_2 + x_3$
Subject to $x_1 + 2x_2 + 3x_3 \leq 90$ (constraint for operation 1)
 $2x_1 + x_2 + x_3 \leq 60$ (constraint for operation 2)
 $3x_1 + x_2 + 2x_3 \leq 80$ (constraint for operation 3)

				3	4	1	0	0	0
Programme Profit Quantity				-----					
				x1	x2	x3	s1	s2	s3
	x2	4	40	0	1	10/6	4/6	-1/3	0
	x1	3	10	1	0	-1/3	-1/3	2/3	0
	s3	0	10	0	0	8/6	8/6	-10/6	1
	(Cj - Zj)			0	0	-28/6	-10/6	-2/3	0

Find the solution, maximum profit, idle capacity and the loss of total contribution of every one unit reduced from the right hand side of the constraints.

Write the dual of the given problem and give the initial Simplex table

4. The budgeted data relating to two products manufactured by a company for a month are as under :

	Product A	Product B
Selling price	300	200
Variable manufacturing cost	160	60
Sales commission	60	40

Each unit of product incurs costs in the company's two departments P and Q. the total capacity available for the month under review is budgeted to be 1,400 hours in department P and 2,000 hours in department Q. The capacity costs amount to ` 14,000 and ` 20,000 respectively per month for P and Q irrespective of the level of usage made of it. The number of hours required in each of these departments to complete one unit of output is as under :

	A	B
Department P	2	4
Department Q	5	4

The maximum output which the company can sell in the month is restricted to 400 units of either of the products. You are required to formulate the Linear Programming (LP) model and solve it graphically to determine the optimal product mix and profit. Write the dual of the given problem.

5. Let us assume that you have inherited `1,00,000 from your father-in-law that can be invested in a combination of only two stock portfolio, with the maximum investment allowed in either portfolio set at `75,000. The first portfolio has an average rate of return of 10%, whereas the second has 20% In terms of risk factors associated with these portfolios. The first has a risk rating of 4 (on a scale from 0 to 10), and the second has 9. Since you wish to maximize your return, you will not accept an average rate of return below 12% or a risk factor above 6. Hence, you then face the important question. How much should you invest in each portfolio? Formulate this as a linear Programming. Problem and solve it by Graphic Method.
6. An agriculture has a farm with 125 acres. He produces Radish, Muttar and Potato. Whatever he raises is fully sold in the market. He gets ` 5 for Radish per kg. ` 4 for muttar per kg. & ` 5 for potato per kg. The average yield is 1,500 kg. of Radish and 1,800 kg. of Muttar per acre and 1,200kg. of Potato per acre. To produce each 100 kg. of Radish and Muttar and to produce each 80kg. of potato, a sum of `12.50 has to be used for manure. Labour required for each acre to raise the crop is 6 man days for Radish and Potato each and 5 man days for Muttar. A total of 500 man days of labour at the rate of `40 per man day are available. Formulate this as a LPP model to maximize the Agriculturist's total profit.
7. The owner of Fancy Goods Shop is interested to determine, how many advertisements to release in the selected their magazines A, B, and C. His main propose is to advertise in such a way that total exposure to principle buyer of his goods is maximized. Percentages of reader for each magazine are known. Exposure in the number of advertisements released multiplied by the number of principle buyers. The following data are available:

	Magazines		
Particulars	A	B	C
Readers	1.0 lakh	0.6 lakh	0.4 lakh
Principle Buyers	20%	15%	8%
Cost per Advertisement	8,000	6,000	5,000

The budget amount is at the most `1 lakh for the advertisements. The owner has already decided that magazine A should have no more than 15 advertisements and that B and C each gets at least 8 advertisement. Formulate a LPP Model for this problem.

8. A Mutual Fund Company has ` 20 lakhs available for investment in Government Bonds, blue chip stocks, speculative stocks and short-term deposits. The annual expected return and risk factor are given below:

Type of investment	Annual Expected Return (%)	Risk Factor (0 to 100)
Government Bonds	14	12
Blue Chip Stocks	19	24
Speculative Stocks	23	48
Short-term Deposits	12	6

Mutual fund is required to keep at least ` 2 lakhs in short-term deposits and not to exceed average risk factor of 42. Speculative stocks must be at most 20 percent of the total amount invested. How should mutual fund invest the funds so as to maximize its total expected annual return? Formulate this as a Linear Programming Problem. Do not solve it.

10. The Delhi Florist Company is planning to make up floral arrangements for the upcoming festival. The company has available the following supply of flowers at the costs shown:

Type	Number available	Cost per flower
Red roses	800	` 0.20
Gardenias	456	` 0.25
Carnations	4,000	` 0.15
White roses	920	` 0.20
Yellow roses	422	` 0.22

These flowers can be used in any of the four popular arrangements whose makeup and selling prices are as follows:

Arrangement	Requirements	Selling price
Economy	4 red roses 2 gardenias 8 carnations	` 6
May time	8 white roses 5 gardenias 10 carnations 4 yellow roses	` 8
Spring colour	9 red roses 10 carnations 9 white roses 6 yellow roses	` 10
Deluxe rose	12 red roses 12 white roses 12 yellow roses	` 12

Formulate a linear programming problem which allows the florist company to determine how many units of each arrangement should be made up in order to maximize profits assuming all arrangements can be sold.

The formulated LP problem is not required to be solved.

11. A company produces two products x_1 and x_2 with respective unit contribution of ` 8 and ` 6.

Each product passes through machining operations in two centres, MI and MII, whose capacities are limited to 60 and 48 hours respectively with corresponding slack variable S_1 and S_2 .

The following table gives the values for an iteration under the simplex method for maximizing the contribution:

	x1	x2	S1	S2	
Basic Variables					
X1	1	0	1/3	-1/6	(MI constraint)
X2	0	1	-1/6	1/3	(MII constraint)

You are required to:

i) Valuate if this iteration represents the optimal solution.

Find out what will be the optimum contribution. (May, 2010 (old), 7 marks)

Where You Stand ?

Formulation of the Problem

1. In a Chemical industry two products A and B are made involving two operations. The production of B also results in a by-product C. The product A can be sold at a profit of ₹ 3 per unit and B at a profit of ₹ 8 per unit. The by-product C has a profit of ₹ 2 per unit. Forecasts show that up to 5 units of C can be sold. The company gets 3 units of C for each unit of B produced. The manufacturing times are 3 hours per unit for A on each of the operation one and two and 4 hours and 5 hours per unit for B on operation one and two respectively. Because the product C results from producing B, no time is used in producing C. The available times are 18 hour and 21 hours of operation one and two respectively. The company desires to know that how much A and B should be produced keeping C in mind to make the highest profit. Formulate LP model for this problem.

2. A firm buys castings of P and Q type of parts and sells them as finished product after machining, boring and polishing. The purchasing cost for castings are ₹ 3 and ₹ 4 each for parts P and Q and selling price are ₹ 8 and ₹ 10 respectively. The per hour capacity of machines used for machining, boring and polishing for two products is given below :

Parts		
Capacity (per hour)	P	Q
Machining	30	50
Boring	30	45
Polishing	45	30

The running costs for machining, boring and polishing are ₹ 30, ₹ 22.5, and ₹ 22.5 per hour respectively.

Formulate the linear programming problem to find out the product mix to maximize the profit.

3. Transport Ltd. provides tourist vehicles of 3 types- 20 seater van, 8-seater big cars and 5 seater small cars. These seating capacities are excluding the drivers. The company has 4 vehicles of the 20 seater van type, 10 vehicles of the eight seater big car types and 20 vehicles of the 5 seater small car types. These vehicles have to be used to transport employees of their client company from their residences to their offices and back. All the residences are in the same housing colony. The offices are at two different places, one is the Head office and the other is the Branch. Each vehicle plies only one round trip per day, if residence to office in the morning and office to residence in the evening. Each day, 180 officials need to be transported in Route I (from residence to head office and back) and 40 officials need to be transported in Route II (from residence to Branch Office and back). The cost per round trip for each type of vehicle along each route is given below.

You are required to formulate the information as a linear programming problem, with the objective of minimizing the total cost of hiring vehicles for the client company, subject to the constraints mentioned above. (only formulation is required. Solution is not needed).

Figures- ₹/round trip	20 seater vans	8 seater big cars	5 seater small cars
Route I: Residence: Head office and Back	600	400	300
Route II: Residence-Branch Office and Back	500	300	200

Simplex Method

4. Noah's Boats makes three different kinds of boat. All can be made profitably in this company, but the company's monthly production is constrained by the limited amount of labour, wood and screws available each month. The director will choose the combination of boats that maximizes his revenue in view of the information given in the following table:

Input	Row Boat	Canoe	Kayak	Monthly Availability
Labour (Hours)	12	7	9	1,260 hours
Wood (Board feet)	22	18	16	19,008 board feet
Screws (Kg.)	2	4	3	396 Kg.
Selling price (in Rupees)	4,000	2,000	5,000	

- (i) Formulate the above as a linear programming problem.
(ii) Solve it by the Simplex method.

From the optimal table of the solved linear programming problem answer the following questions:

- (iii) How many boats of each type will be produced and what will be the resulting revenue?
(iv) Which, if any, of the resources are not fully utilized? If so, how much of spare capacity is left?
(v) How much wood will be used to make all of the boats given in the optimal solution?
(vi) State the dual of the formulated linear programming problem.

Answer: (ii) The solution is given by: $x_1 = 12$, $x_2 = 0$ and $x_3 = 124$.

- (iii) The company should produce 12 Row boats and 124 Kayak boats only. The resulting revenue will be:

$$= 4,000 \times 12 + 2,000 \times 0 + 5,000 \times 124$$

$$= 48,000 + 6,20,000 = 6,68,000$$
- (iv) Wood is not fully utilized. Its spare capacity is 16,760 board feet.
- (v) The total wood used to make all of the boats given in the optimal solution is $(22 \times 12 + 16 \times 124)$ board feet = 2,248 board feet.

5. Three grades of coal A, B and C contains phosphorus and ash as impurities. In a particular industrial process, fuel up to 100 ton (maximum) is required which could contain ash not more than 3% and phosphorus not more than 0.03%. It is desired to maximize the profit while satisfying these conditions. There is an unlimited supply of each grade. The percentage of impurities and the profits of each grades are as follows:

Coal	Phosphorus (%)	Ash (%)	Profit in ` (Per ton)
A	0.02	3.0	12.00
B	0.04	2.0	15.00
C	0.03	5.0	14.00

You are required to formulate the Linear Programming (LP) model to solve it by using simplex method to determine optimal product mix and profit.

[Answer: The optimal solution is $X_1 = 40$, $X_2 = 40$ & $X_3 = 20$ with maximum $Z = 1,360$

6. A gear manufacturing company makes two types of gears – A and B. Both gears are processed on 3 machines, Hobbing M/c, Shaping M/c and Grinding M/c. The time required by each gear and total time available per week on each M/c is as follows:

Machine	Gear (A) (Hours)	Gear (B) (Hours)	Available Hours
Hobbing M/c	3	3	36
Shaping M/c	5	2	60
Grinding M/c	2	6	60
Other data:			
Selling price (₹)	820	960	
Variable cost (₹)	780	900	

Determine the optimum production plan and the maximum contribution for the next week by simplex method. The initial table is given below:

			Cj	40	60	0	0	0
			Qty.					
Cj	Variable			X1	X2	X3	X4	X5
0	X3	36		3	3	1	0	0
0	X4	60		5	2	0	1	0
0	X5	60		2	6	0	0	1

[Answer: Optimum Z = 660 with X1 = 40 and X2 = 60]

General Problem

7. The following matrix gives the unit cost of transporting a product from production plants P_1 , P_2 & P_3 to destinations W_1 , W_2 and W_3 . Plants P_1 , P_2 and P_3 have a maximum production of 65, 24 and 111 units respectively and destinations D_1 , D_2 and D_3 must receive at least 60, 65 and 75 units respectively.

To \ From	D_1	D_2	D_3	Supply
P_1	400	600	800	65
P_2	1,000	1,200	1,400	24
P_3	500	900	700	111
Demand	60	65	75	200

You are required to formulate the above as a linear programming problem. (only formulation is needed. Please do not solve).

8. A firm produces three products A, B, and C. It uses two types raw materials I and II of which 5,000 and 7,500 units respectively are available. The raw material requirements per units of the products are given below:

Raw Material	Requirement per unit of Product		
	A	B	C
I	3	4	5
II	5	3	5

The labour time for each unit of product A is twice that of product B and three times that of product C. The entire labour force of the firm can produce the equivalent of 3,000 units. The minimum demand of the three products is 600, 650, and 500 units respectively. Also the ratios of the number of units produced must be equal to 2: 3: 4. Assuming the profit per units of A, B and C as ₹ 50, ₹ 50 and ₹ 80 respectively.

Formulate the problem as a linear programming model in order to determine the number of units of each product, which will maximize the profit.

9. The costs and selling prices per unit of two products manufactured by a company are as under:

Product	A (₹)	B (₹)
Selling Price	500	450
Variable costs:		
Direct Materials @ ₹ 25 per kg.	100	100
Direct Labour @ ₹ 20 per hour	80	40
Painting @ ₹ 30 per hour	30	60
Variable overheads	190	175
Fixed costs @ ₹ 17.50/D.L.Hr.	70	35
Total costs	470	410
Profit	30	40

In any month the maximum availability of inputs is limited to the following:

Direct Materials	480 kg.
Direct Labour hours	400 hours
Painting hours	200 hours

Required:

(i) Formulate a linear programme to determine the production plan which maximizes the profits by using graphical approach.

(ii) State the optimal product mix and the monthly profit derived from your solution in (i) above.

(iii) If the company can sell the painting time at ₹ 40 per hour as a separate service, show what modification will be required in the formulation of the linear programming problem. You are required to re-formulate the problem but not to solve.

10. An advertising firm desires to reach two types of audience customers with annual income of more than ₹ 40,000 (target audience A) and customers with annual income of less than ₹ 40,000 (target audience B). The total advertising budget is ₹ 2,00,000. One programme of T.V. advertising costs ₹ 50,000 and programmes of Radio advertising costs ₹ 20,000. Contract conditions ordinarily require that there should be at least 3 programmes on T.V. and the number of programmes on Radio must not exceed 5. Survey indicates that a single T.V. programme reaches 7,50,000 customers in target audience A and 1,50,000 in target audience B. One Radio programme reaches 40,000 customers in target A and 2,60,000 in target audience B.

Formulate this as a linear programming problem and determine the media mix to maximize the total reach using graphic method.

11. A manufacturer produces three products Y_1 , Y_2 , Y_3 from three raw materials X_1 , X_2 , X_3 . The cost of raw materials X_1 , X_2 and X_3 is ₹ 30, ₹ 50 and ₹ 120 per kg respectively and they are available in a limited quantity viz. 20 kg of X_1 , 15 kg of X_2 and 10 kg of X_3 . The selling price of Y_1 , Y_2 and Y_3 is ₹ 90, ₹ 100 and ₹ 120 per kg respectively. In order to produce 1 kg of Y_1 , $\frac{1}{2}$ kg of X_1 , $\frac{1}{4}$ of X_2 and $\frac{1}{4}$ kg

of X_3 are required. Similarly to produce 1 kg of Y_2 , $3/7$ kg of X_1 , $2/7$ kg of X_2 and $2/7$ kg of X_3 and to produce 1 kg Y_3 , $2/3$ kg of X_2 and $1/3$ kg of X_3 will be required.

Formulate the linear programming problem to maximize the profit.